The Students' Mathematical Thinking Ability In Solving The Program for International Student Assessment(PISA) Standard Questions

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Abstract- The aim of the research is to describe and analyze the students' mathematical thinking ability in solving the PISA standard questions. This research uses qualitative method. There are 8 of the Junior High School students in Jember who are high order thinking involved and tested with the questions based *on* Program for International Student Assessment(PISA) standard. The qualitative data are obtained through interview and observation. The data were analyzed through the application of data reduction and data presentation, leading to the study's findings and conclusion. The findings of the research are: (1) Using mathematical thinking in solving the PISA Standard questions can improve the reasoning ability of the students, (2) Identifying the level of the students' mathematical thinking is needed to help the students in developing their mathematical thinking ability and to improve the level of PISA acquisition scores, (3) The students' mathematical thinking ability is fairly good, however still needs to be guided by the teachers in order to strengthen their understanding, especially in the Convincing process.

Keywords:ability, mathematical thinking, PISAstandard.

Introduction

Thinking is a kind of mental activity that produces new representation through the transformation of the information that involves complex interaction among mental attributes such as assessment, abstraction, imagination, and problem solving(Siswono,2018). It indicates that thinking is an activity undertaken individually. Tall (1991) stated that mathematical thinking is related to students' existing concepts and learning from procedural association. In order to attain this level of mathematical thinking, Tall stated that a concept is specific knowledge by which individuals can explain or practice to acquire theory, formulae and mathematical apprehension (Evitts, 2004). Mathematical thinking not only requires knowledge creation but also knowledge enhancement (Tall,1991, 1995). In mathematical thinking, a person needs to have: 1) deep knowledgeabout mathematics, 2) the ability to generalize, 3) knowledge of strategies that arewill be used (Stacey, 2005). This is also in line with the opinion of Mason (1982) whomention 3 factors that affect the effectiveness of one's mathematical thinking, namely 1)The ability to solve problems, 2) emotional control and psychology inthe process of solving problems, 3) understanding of mathematical concepts togetherits application.

Focusing on specific processes of mathematical thinking practices , needed students to be successful mathematics students. They were based on five key areas 1) Representation, 2) Reasoning and Proof, 3) Communication, 4) Problem Solving, and 5) Connections. If these look familiar, it is because they are the five process standards from the National Council of Teachers of Mathematics(NCTM, 2000). Mathematical thinking itself is an individual activitybased on personal experience and can also focus on associating key ideas owned (Stacey, 2010). Associating these ideas will certainly be related toasking questions related to what is known, what is desired, andhow to solve it.Thus, the most important ability which needs to be developed for the students is mathematical thinking ability. This is because thinking mathematically and using mathematics can help the students to face their

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real life situations (Cai, 2002; Embong et al, 2010; and Wood et al, 2006). Furthermore, the result of Parmjit's research (2018) shows that the implementation of heuristic has a positive effect on the development of the students' mathematical thinking. Besides, the thing that should be concerned in the process of learning mathematics is the students need to be trained in the ways of thinking and reasoning to make a conclusion through some activities such as investigating, exploring, experimenting, showing similarities and differences, and understanding the consistencies and inconsistencies.

One of the most fundamental goals of mathematics education is being able to use mathematicalthinking in solving problems (Stacey, 2006; Shahrill, et al. 2018). Therefore, teachers should supportthe development of students' mathematical thinking. For this purpose, accessing students' thinking when students solve problems is essential to get information about students' potential and alsodifficulties. Suchminformation is important as a consideration for teachers to choose and prepareappropriate teaching strategies and/or materials. Despite the importance of accessing students'thinking process is not easy because what teachers can directly access are student learning written andoral activities. As mentioned by von Glaserfeld (1995), teachers do not have direct access to students'mathematical thinking. What teachers can do is accessing the evidences of students' mathematicalthinking, such as students' works.

Mathematical thinking ability is an important ability to be mastered by the students. It is supported by Oers (2010) that mathematical thinking ability is a way to master the mathematics itself. Therefore, it can be stated that mathematical thinking ability must be owned by the students in order to establish their mathematical learning ability, also to achieve the learning goals in general. Farther, it has a positive impact for the students on the ways of thinking in dealing with the problems in various aspects in their real life situation. In addition, the similar idea is highlighted by Gerdes (2000) that humanity will lose enormous resources from science if mathematical thinking ability is not a major part of learning, for today and for the future.

Conceptually, the issue related to the development of mathematical thinking ability has been stated in Permendikbud RI No. 24 Year 2016 about Core Competence and Basic Competence for the Primary and Secondary Education unit. In addition, especially for mathematics, the dimension of mathematical thinking is stated in the series of learning competencies that are hierarhically become one specific ability that must be mastered by the students. However, the directions are still need to be given to students in order to help them in improving and developing their mathematical thinking ability.

The ability to think mathematically and to use mathematical thinking to solve problems is an important goal of schooling. In this respect, mathematical thinking will support science, technology, economic life and development in an economy. By teaching specific strategies of problem solving to students would notonly increase student confidence as they learned to work problems,cause it through thesteps of reasoning, but would also require higher level thinking and have real world applications.So students who reasoned and solved problems were much better equipped to function intoday's society than those who did not have.

Furthermore, mathematical thinking can be viewed as a way of understanding mathematical problems bycompiling various sources of study of mathematical objects. Mason, Burton & Stacey (2010) reinforces with theirexpression that mathematical thinking is a process and is a very complex activity, so to understand it can be doneby giving an example. The process of mathematical thinking describes sequences in thinking activities, forexample if in a student arises a problem to be solved, a schematic/chart is still unclear. In addition, thescheme/chart is solved or correlated, and compared carefully to a conclusion (Mustafa, 2019).To understand the students' mathematical thinking ability, the one of the efforts that can be done is give a test that accommodates students' ability in solving math problems.

Increasingly, governments are recognizing that economic well-being in a country is underpinned by strong levels of what has come to be called 'mathematical literacy' (PISA, 2006) in the population. Mathematical literacy is a term popularised especially by the OECD's PISA program of international assessments of 15 year old students. Mathematical literacy is the 39 ability to use mathematics for everyday living, and for work, and for further study, and so the PISA assessments present students with problems set in realistic contexts. The framework used by PISA shows that mathematical literacy involves many components of mathematical thinking, including reasoning, modelling and making connections between ideas. It is clear then, that mathematical thinking is important in large measure because it equips students with the ability to use mathematics, and as such is an important outcome of schooling. At the same time as emphasising mathematics

The most important ability that needs to be taught to students regarding the ability to think and make decision independently is the mathematical thinking ability (Masami, 2012). Thus, it is important for teachers to lead the students in developing their mathematical thinking ability to master mathematics. As a mathematics teacher, it was important to identify and clearly communicatemathematical thinking ability in the classroom. The students had multiple opportunities to discuss what makes good mathematical thinking ability and were able to viewmathematical thinking ability through solving problem of PISA standard questions, so good mathematical thinkers wouldemerge in the classroom.

Mathematical thinking ability can be developed through PISA standard questions by paying attention to the level of the students' mathematical thinking ability. It is because the framework used by PISAshows that mathematical literacy involves the components of mathematical thinking such as, reasoning, modeling, and making relationships between ideas. The efforts to identify the development of the students' mathematical thinking level is important to be conducted. It is because they will always use their mathematical thinking ability during learning mathematics. Besides, the mathematical thinking ability itself will has a great connection with the content and mathematical methods. Farther, associated to the main goal of this study, the results of this study are expected to provide the information toward the students' mathematical thinking ability in solving thePISA standard questions. Therefore, it can give a good contribution in developing the students' mathematical thinking ability in order to improve the level of PISA acquisition scores, then it can give a good advancement for the quality of the mathematics literacy in Indonesia.

The purpose of this research then, was to describe and analyze the students' mathematical thinking ability in solving the PISA standard questions. The following research questions:What will happen to mathematical thinking ability of student when students solving problem of PISA standard questions?

Review Of Literature

Problem Solving as a Process

Several researchers noted using problem solving as a process in order to promote higher level thinking and reasoning. Many mention some common skills in problem solving I wanted to pursue. According to Segurado (2002) good problem solvers are confident in their abilities.

It is possible to provide students of this school level mathematical experience of doing investigations. Students are able to approach the tasks and move in the direction of becoming confident in their abilities, of enlarging their ability to solve and formulate problems and of communicating and reasoning mathematically. (p. 72)

Costa and Kallick (2000) say those good at problem solving are risk takers. Students who practice what they call responsible risk taking show a willingness to try out new strategies or techniques and are willing to test new hypotheses with an attitude of "What's the worst thing that can happen? We'll only be wrong?" Costa and Kallick also list persistence among the skills of those good at the problem solving process. They say in order to be successful problem solvers, students must not give up when encountering a difficult problem, even if they are not used to such struggle.

Persistent students have systematic methods of analyzing a problem. They knowhow to begin, what steps must be performed, and what data need to be generatedand collected. They also know when their theory or idea must be rejected so theycan try another…If the strategy is not working, they back up and try another. (p.22)

Fan and Zhu (2007) talk about a framework for problem solving modified from Polya'sproblem-solving model and published in a syllabus by the Ministry of Education in 1990. Its listincludes developing a plan, carrying out the plan and/or modifying the plan if necessary andending with seeking alternative solutions and checking for reasonableness. Students good atproblem solving do all of these things.Costa and Kallick (2000) say that as students increase in their problem solving ability,they become more flexible in their thinking. They consider, express or paraphrase other points ofview, can state several ways of solving the same problem, and evaluate the merits of more thanone course of action. Students who have this habit of mind in place become systems thinkers.They analyze and scrutinize parts, but also shift their perspective to the big picture.

The Australian Mathematics Education Program (AMEP), established by the CurriculumDevelopment Centre (CDC), in its first national statement of basic mathematical skills andconcepts (CDC, 1982) states,Problem solving is the process of applying previously acquired knowledge in newand unfamiliar situations. Being able to use mathematics to solve problems is amajor reason for studying mathematics at school. Students should have adequatepractice in developing a variety of problem solving strategies so they haveconfidence in their use. (p. 3)Good problem solvers do just that. When given an unfamiliar problem, they know what to do andcan switch strategies because they have an unofficial list of problem solving strategies to callupon.

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Successful problem solvers are agile users of what Schoenfeld (1994) calls the tools andlogic of mathematics.That ability is improved through the solving of "good problems."Schoenfelddefinesagood problem:Goodproblems can introduce students to fundamental ideas and to theimportance of mathematical reasoning and proof. Good problems can serve asstarting points for serious explorations and generalizations. Their solutions canmotivate students to value the processes of mathematical modeling andabstraction and develop students' competence with the tools and logic ofmathematics.(p.60). So, to be good at problem solving a student must exhibit the following:1) showconfidence in solving problems; 2)demonstrate persistence when encountering adifficultproblem and refuses to give up; 3) when given an unfamiliar problem, knowswhat to do and canswitch strategies if one is not working; and 4) has an unofficial list of problemsolvingstrategiestocalluponwhensolvingproblems.

The Mathematical Thinking

Thinking is typically defined as the means used by individuals to improve their understanding of, and exert some control over, their environment (Burton, 1984, p. 36). To do this, mathematical thinking lies on particular means such as different registers or representations that can be recognized as arising from or pertaining to the study of mathematics. Different registers or representations can be interpreted as: (a) modes of representation (Bruner, 1966), (b) modes of operation (Hughes-Hallett, 1991), and (c) worlds of mathematics (Tall, 2004). These three theories are not meant to be exhaustive but represent a useful set of core theories that cut across all mathematical domains. Here it is necessary to mention that mathematical thinking aspects become more sophisticated when the individual becomes more experienced (Giraldo, Tall, & Carvalho, 2003)

Problem solving, reasoning and communication are required for mathematical thinking (Suzuki, 1998). Mathematical thinking is defined as the ability to judge the data, status and objects with mathematical logic (Tasdemir and Salman, 2016). Mathematical thinking is the power that enables understanding and comprehension, and that reveals knowledge and skills to solve problems (Katagiri, 2004). Mathematical thinking has a unique function separate from general frame of thinking and continuous cognitive processes (Argle, 2012). Mathematical thinking, which is defined as the scientific skill that should be developed in mathematics classes, directs the student to learn mathematics as an information process (Katagiri, 2004). In the most general sense, mathematical thinking can be defined as using mathematical techniques,concepts, and methods, directly or indirectly, in theproblem-solving process.

The aspect of mathematical thinking ability that induces and ways of thinking systematically, also using symbols (Mamoon, 2012). Humans have processes in intelligence that are different from the ability of every human being to have applications in his intelligence in solving problems and creating objects (Gardner, 1993; Stacey, 2011).). Ability to think mathematically is logical thinking and analysis as reasoning (Devlin, 2012). Mathematical thinking indicator to be; interpret, analyze, evaluate and infer (Facione, 1994). Teachers involved support student learning should be aware of potential gaps in students' knowledge of the core components of mathematical reasoning, such as estimation, probability, sampling and algebra (Gerry & Judith, 2004).Mason (2011) adds that there are 3 factors that influence the effectiveness of one's mathematical thinking, they are: 1) an ability to solve the problems, 2) an ability to control the emotion and psychology in the process of problem solving, then 3) an understanding toward the concept of mathematics and its application. The association of the ideas will certainly be related to the submission of questions related to what is known, what is desired, and how to solve it.

Furthermore, Stacey (2011) higlights some processes that are passed in thinking mathematically, such as: 1) specializing, 2) generalizing, 3) conjecturing, and 4) convincing. Moreover, it needs to be noticed that the ability to think mathematically and to use the mathematical thinking in solving the problem is an important goal that the schools want to achieve. In this case, mathematical thinking will support science, technology, economic life, and economic development within. Then, the government also increasingly recognizes that economic prosperity in a country is sustained by a strong level of what is then called 'mathematical literacy' (PISA, 2006).

The following were four indicators of mathematical thinking recommended by Stacey showed in table 1.

Table 1. The Indicators of the Students' Mathematical Thinking Ability

athematical Thinking Proce	Indicators
;	he problems
	nd trying various possible strategies
g	he ideas made
	he scope of the results obtained
g	; the similar ideas
	the reasons why the reasults can appear
	ttern of known results
	opposite pattern from that has been formed

Mathematical literacy is a term popularized primarily by PISA OECD program, an international assessment of 15-year-old students. The definition of mathematical literacy is the ability to use mathematics for everyday life, works, and for further study, thus the PISA assessment presents the students with problems arranged in a realistic context. Moreover, the framework used by PISA shows that mathematical literacy involves many components of mathematical thinking such as; reasoning, modeling, and making relationships between ideas.

Students are taught mathematics not only to get the high scores in examinations, but they are expected to be able to apply their knowledge to face and solve the problems in real life situations. Organization for Economic Co-operation and Development (OECD) and Partnership for 21st Century Skills report that the modern world needs more than knowledge, thus the reasoning and problem solving abilities are the demands ofPISA questions (Wardhani, 2005). PISA(Programme for International Student Assessment) is an international organization that functions to assess the students' mathematical literacy. According to PISA, the students are considered as capable if they can apply their previous knowledge to solve the unknown new problem. This ability is commonly known as the ability to think mathematically. As a result, it is necessary to develop the mathematical thinking ability through PISA questions.

PISA develops six categories of the students' mathematical abilities that show the students' cognitive abilities. The levels of mathematical ability according to PISA are presented on the following table, table 2.

Table 2. Levels of the Students' Mathematical Thinking Ability According to PISA

Levels	Descriptions
1	s are able to use their knowledge to solve the routine problems, and able t the general context.
2	s are able to interpret the problems and solve them by using formulas.
3	s are able to perform the procedures in answering the questions, and able to ving strategies.
4	s are able to work effectively with the models, able to choose and integrate tl ons, then connect them with the real life situations.
5	s are able to work with models for complex situations, then able to sol
6	s are able to use their reasoning ability to solve the mathematical problems, to , then to communicate the findings.

Resource: Johar 2012

The above table provides the explanations toward the levels of mathematical ability developed by PISA. It can be seen that the assessment of mathematical literacy conducted by the PISA study consists of 6 levels. The mathematical literacy questions for level 1 and 2 are included into the lower-scale group that measure the reproductive competence. The questions are arranged based on the familiar context for the students with the simple mathematical operations. Moreover, the mathematical literacy questions for level 3 and 4 are included into the medium-scale group that measure the connection competence. These kind of questions need the students' interpretation since the given situations are unknown or never even experienced by them. Meanwhile, the mathematical literacy questions for level 5 and 6 are included into the high-scale group that measur the reflection competence. These questions require a high level interpretation with the context that is totally unexpected by the students.

Method

General Backround of Research

This research uses qualitative method. The qualitative data are obtained through interview and observation. The data were analyzed through the application of data reduction and data presentation, leading to the study's findings and conclusion.

Participant in Research

The research is conducted in SMP 01 Rambipuji at theIX class, Jember, East Java, Indonesia.There are 8 of the Junior High School students in Jember who are high order thinking involved and tested with the questions based onProgram for International Student Assessment(PISA) standard. The place of the research is selected to examine the students' mathematical thinking ability in solving the PISA standard questions.Subjects of the research are

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chosen by checking the answers from all of the students in solving the PISA standard questions based on the levels of the students' mathematical thinking ability according to PISA, they are Low Order Thinking (C1-C3), and High Order Thinking (C4-C5). The Low Order Thinking is categorized for the students who cannot answer the PISA standard questions for level 4, 5, and 6 correctly. Meanwhile, the High Order Thinking is categorized for the students who can answer the PISA standard questions for level 4, 5, and 6 correctly. Especially for this research, there are three students randomly selected from the High Order Thinking (C4-C5) as the research subjects.

Instrumen

The instruments used in this research were 1) a test which comprised three levels PISAProblems. They were PISA standard questions for level 4, 5, and 6. The PISA standard questions are the math questions that have been developed by the researcher based on the students' context 2) an observation sheet for observing mathematical thinking during PISA problem-solving, and 3) a guided semi-structured interview for confirming the behavioural tendencies of the participants when solving given problems. The content validity of the test, the observation sheet, and the guided semi-structured interview were addressed by involving various professors across related disciplines. This consultation resulted in a series of substantive changes to the test, observation sheet, and semi-structured interview

Data Collection Methods

Data in this research were collected through testing, direct observation, and interviewing. The test was given to determine the characteristics of the research subjects' responses and attitudes; it employed the indicators of mathematical thinking. All the subjects were required to solve three problems of PISA Standar questions; they werethree level of PISA standar questions. During the test, the researchers also observed and recorded the participants' behaviour when solving PISA standar questions then classified their mathematical thinking using an observation sheet. The last data collection method was the in-depth interview. Each interview was recorded using a video, and the participants' written test results were collected. To ensure the validity and reliability of the research, data analysis was performed by triangulation: the data from the direct observations was displayed and compared with the participants' written test data and the interviews (Creswell, 2010). This was carried out to determine the participants' behavioural tendencies when solving PISA standar problems based on the indicators of mathematical thinking. The data resulting from the triangulation were analyzed.

Data Analysis

The data were analyzed through the application of data reduction and data presentation,
leading to the study's findings and conclusion. In the first stage, the data reduction dealt with including pertinent variables which were in line with the research objectives and excluding unnecessary variables. The second stage, data presentation, was related to classifying the reduced data into two groups, namely, (1) the thinking tendencies of the students in the solution of PISA standar problems based on the indicator of mathematical thinkingand(2)the behaviour of the students in the solution of PISA standar problems based on the indicator of mathematical thinking. The final stage entailed drawing conclusions based on the findings and the data presentation.

Findings

Categorization of the Students' Mathematical Thinking Ability

Related to the study that has been conducted in the school, the researcher will describe the findings collected from all of the research subjects, that is 8 students. This study is based on all of the subjects who had answered the PISA standard questions and the interviews at the predetermined time. The scores result obtained from the students' answers in solving the PISA standard questions also the interviews, are then used in determining the level of the students' mathematical thinking ability based on the levels of PISA presented in table 4. Moreover, the researcher categorized the students' mathematical thinking levels based PISA standard.

Hereinafter, the qualitative analysis is conducted on the students' answers and the interview results to identify the students' mathematical thinking ability after the tests are given. The test results show that there are 8 students at the High Order Thinking (C4-C5). Therefore, the researcher takes 3 samples randomly from the High Order Thinking (C4-C5) category. In this case, the researcher takes the mathematical thinking indicators suggested by Stacey as stated on the table 1, they are: 1) Specializing; 2) Generalizing,3)Conjecturing, and4)Convincing. Moreover, the interview result and its analysis are presented below:

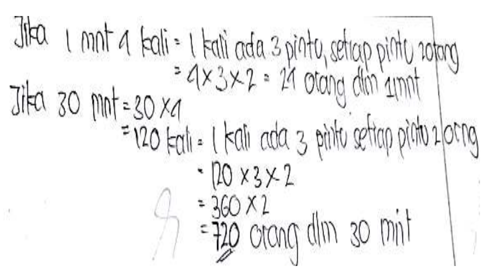
The Mathematical Thinking of the students When Solving PISA problem 1 (PISA Standar Question level 4)

Aston Hotel is a classy hotel in Jember. The hotel has a turnstile with 3 door wings. The diameter of the door is 2 meters (200 cm). The following picture shows the turnstile in that hotel.

If the door rotate 4 times every one minute, and every one door space is for 2 people. How many people can enter thebuilding through the door for 30 minutes?

Figure 1 Problem PISA 1 (PISA Standard Question Level 4

Problem PISA 1(PISA Standard Question Level 4) is chosen according to the indicators of PISA questions level 4, which is to involve the students in working effectively with the models, then in choosing and integrating the different representations, also connecting them with the real world. In finishing the problem PISA 1, subject RB do not make mistake in determining the solution to the problems given. The subject identifies the problem correctly by giving the information in the question also the objective to be achieved. The answer sheet also shows that he can work effectively with the model, and can choose and integrate the different representations, also can connect it to the real world. In addition, through the interview session, there are some important information that are collected. The subject gives the reason why he integrates the different representations and relates them to the real world. According to him, those activities help him in solving the problems fastly. Moreover, when the researcher asks what if the time is extended or shortened, he tells that the results will be bigger or smaller.



Translate version

If 1 minute four times cycles = one times there are three doors, every door there are 2 people

= 4 x 3 x 2 = 24 people in one minute

If 30 minutes = 30 x4

= 120 times = 1 times there are three doors, every door there are two people

= 120x 3 x 2

Figure2 The Result of the Students' Work(RB)

The above activities are conducted by the researcher to see the conjecturing part by analogizing the similar cases, then to check the convincing part by looking for the reasons why the results could appear. Referred to the answers given by the subject, it is known that the conjecturing and convincing part are also detected in the subject. Subsequently, the researcher gives a question related to the convincing part, how is the solution to be generally accepted, the subject also gives a correct answer. Thereupon, when the researcher gives the opposite case, the subject can also answer correctly. Thus, it can be stated that for convincing part the subject is able to make an opposite pattern that has been formed. In this solving problem the students planned well. This is in accordance with Suarsana's findings (2019) that the right planning can guide the students solve the problems in amore structured way, which implies to better students' problem-solving in mathematics

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The Mathematical Thinking of the Students When Solving PISA Problem 2 (PISA Standar Question level 5)

> For the *rock* music concert in Jember, in the Kaliwates sub-district there is a rectangular field with the following size: 100 meters length and 50 meters width. The field is prepared for the visitors. The tickets are sold out even many fans stand up. What is the number of the visitors who come to the concert? Write down your reasons!
> a. 2.000 b. 5.000 c. 20.000 d. 50.000 e. 100.000

Figure 3 Problem PISA 2 (PISA Standard Question Level 5)

Problem PISA 2 is chosen based on the indicators of PISA level 5, that is involving the students to work with the models for complex situations and to solve the complex problems. In solving the problem PISA 2, subject AA do not do not make mistake in determining the solution to the problems given. The subject identifies the problem correctly by giving the information in the question also the objective to be achieved. The answer sheet also shows that she/he can work with the models for complex situations and to solve the complex problems. Farther, during the interview session, the researcher asks the reason why 5000 m^2 times 4, the subject explains that 1 m^2 is filled for 4 people. Later on, when the researcher asks what if 1 m^2 is filled for 1 person, the subject adds that it does not make sense since the tickets are sold out and many fans stand up.

Translate version

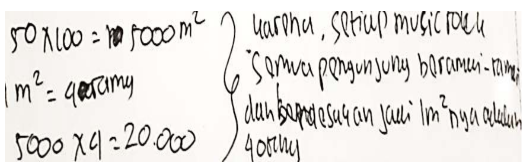


Figure4. The Result of the Students' Work (AA)

All of the activities are conducted by the researcher to check the conjecturing part by analogizing the similar cases, then to check the convincing part by looking for the reasons why the results could appear. Associated to the answers given by the subject, it is known that the conjecturing and convincing part are also detected in the subject. In addition, the researcher gives a question related to the convincing part, how is the solution to be generally accepted, the subject can not give a correct answer, so it can be stated that the convincing part is missing from the subject. Afterwards, when the researcher gives the opposite case, the subject also can not answer correctly. Thus, it can be stated that for convincing part the subject AA is not able to make an opposite pattern that has been formed.

The Mathematical Thinking of the Students When Solving PISA Problem 3 (PISA Standar Question level 6)

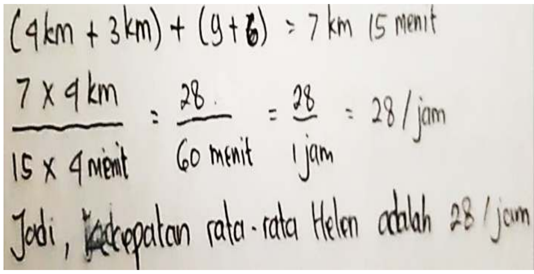
> Helen uses bicycle to the Bedadung river from his house for 4 km and takes 9 minutes. To go home, he takes a short lane, that is for 3 km and takes 6 minutes. What is the average speed done by Helen in km/hour, for the trip to the river and back home?

Figure 4.Problem PISA 3 (PISA Standard Question Level 6)

Problem PISA 3 refers to the indicators of PISA level 6, that is involving the students to use their reasoning ability in solving the mathematical problems, to make a generalization, to formulate and communicate the findings. In solving the problem PISA 3, the subject MG do not do not make mistake in determining the solution to the problems given. The subject identifies the problem correctly by giving the information in the question also the objective to be achieved. The answer sheet also shows that he can use his reasoning ability in solving the mathematical problems. Then, during the interview session, the researcher finds out the reason why the subject summing the two distances and the two times, she/he explains that in that way the average speed can be obtained.

Furthermore, when the researcher asks why she/he multiplied by number 4, the subject answers because the average speed is km/h, it must be multiplied by 4, so that it becomes 60 minutes. However, in the answer sheet the subject forgets to write down the units of km/h, he only writes the units of hours.

Translate version



(4 km + 3 km) +(9+6)= 7 km 15 minutes

$$\frac{7 \times 4\ km}{15 \times 4\ minutes} = \frac{28}{60\ minutes} = \frac{28}{1\ hour} = 28/hour$$

So, the average speed done by

Figure5. The Result of the Students' Work (MG)

Those activities are conducted by the researcher to identify the conjecturing part by analogizing the similar cases, then to check the convincing part by looking for the reasons why the results could appear. Related to the answers given by the subject, it is known that the conjecturing and convincing part are also detected in the subject. Farther, the researcher gives a question related to the convincing part, how is the solution to be generally accepted, the subject also gives a correct answer. On the other hand, when the researcher gives the opposite case, the subject seems confused. So, for the convincing part, it is known that the subject can not make the opposite pattern at all.

Discussion

Based on findings of the study, The subject RB, AA,and MG identify the problem correctly by giving the information in the question also the objective to be achieved. It was mean the student did stage of mathematical thinking specializing. In identifying the problem, the students firstly determine the known elements, giving mathematical terms or symbols, then designing the solution. Mustafa (2019) said that Identify problems, carried out through activities of making perceptions, analyzing and establishing identity.Perception is defined as the act of composing, recognizing, and interpreting sensory information in order toprovide a picture and understanding of the object.The sheet answer of Subject RB and MG also shows that They can use his reasoning ability in solving the mathematical problems When student solvedthe problems they faced and when they ascend top steps ofmathematical thinking, they showed a better performancewhen compared to the other studies. Because, in the stepsof generalization.(Sevinc, 2019). On the other hand, when the researcher gives the opposite case, the subject AA and MG seems confused. So, for the conjecturing and convincing part, it is known that the subject can not analogizing in other case, They can't make the opposite pattern at all.

Based on the result of the study, it is known that mathematical thinking is important for students to have. This is because mathematical thinking is a way to learn math itself (Oers. 2010). Mathematical thinking ability can be developed through the PISA standard questions by paying attention to the levels of the students' mathematical thinking ability. It is because the framework used by PISA shows that the mathematical literacy involves the components of mathematical thinking including reasoning, modeling, and making connections between ideas. The efforts to identify the students' mathematical thinking ability is important to improve the students' mathematical thinking up to the next level since it is used during math activities. Especially for this research, the researcher takes into account the students' responses in solving the PISA standard questions and identifies the students' mathematical thinking processes.

Conclusion

Based on the research findings and discussions abovewhich is based on observations of Eight students, it isconcluded that in solving the PISA standard questions, the mathematical thinking process is needed to understand and solve the problems provided in the questions. Each phase of mathematical thinking has an impact

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on students' ability towards mathematical problems, so they can illustrate the thinking process of the students. Phaseconsists of specializing, generalizing, conjecturing, and convincing.The researcher finds out that the students' mathematical thinking ability is fairly good, but still needed to be guided by the teachers in order to strengthen their understanding, especially in the process of convincing by making the opposite of the pattern that has been formed. Therefore, it is concluded that during the learning process, the teachers can apply the mathematical thinking process through the PISA standard questions for students. It is conducted to facilitate the students in mastering the mathematical concept in school and to lead the students to have a high order thinking ability in learning.

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