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To cite this article: U Farihah 2019 *IOP Conf. Ser.: Earth Environ. Sci.* **243** 012104

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# Student modelling in solving the polynomial functions problems using Geogebra approach

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**Abstract.** The use of technology has crucial influences on mathematical modeling. The purpose of this study was to examine students' approaches to the contextualized problems in the "technology world" of the modeling activity. This research employed a qualitative descriptive study. The subjects of this research were six of the fourth semester students of Mathematics Education Department, State Islamic Institute of Jember, East Java Indonesia academic year 2017-2018 with different degree of visuality; two visual students, two harmonic students, and two non visual students. They solved several tasks involving polynomial models, where students regularly used geogebra. Based on the results of research analysis, it was shown that the visual students skipped much of the algebra work, and rather chose a geometrical approach. While harmonic and non-visual students rather chose the algebraic approach, they skipped much of the geometric work

## 1. Introduction

Technology plays an important role in mathematics education. With the advent of innovative technology, mathematical perception has changed, especially over the past three decades. Improvements in technological tools open the door to the emergence of new methods in learning mathematics. Technology influences mathematical activity [1]. In many ways, technology challenges traditional hierarchy and termination of mathematical topics. It also reshapes the nature and purpose of mathematical representation in mathematical work. It has a clear implication on the context of learning based on the development of a given real world mathematical model. Technology can provide many connections in mathematics, especially supporting holistic development of mathematical understanding [2].

Developing and exploring mathematical models using technological tools reveal new side that goes far beyond the idea of getting more computing or graphics skills in handling mathematical models. Some researchers have suggested that modeling cycles need to be re-conceptualized to integrate the third world that is the technology world [3, 4]. Not only the modeling cycle can be augmented to include a third world where the computer model and computer results are fundamental parts but, most importantly, the impact of digital tools occurs at all stages of the modeling cycle. The formulation of mathematical models and computer models are fused together and the same goes for the application of mathematical models and the implementation of computer models. Calculus learning is often considered problematic. Some studies examined students' errors in studying calculus. Those studies suggested that some of the causes of difficulty in studying calculus are lack of mastery over the concept of calculus [5, 6]. In the learning process of the classroom, the lecturer plays a role in conveying and explaining the material, in order to be understood and mastered by the students. However, it should also be realized that the ability of each student is probably different. This can be



seen from the lack of students involvement in learning activities, non-optimal learning outcomes, and also the small amount of enthusiasm and interest of students in learning activities .

Taking this into account, a suitable method, strategy or instructional media is required to apply. The use of computer-based learning media in learning calculus is very relevant, considering the material characteristics of calculus are abstract mind objects. This is what often causes great difficulty for students to study calculus. In this case, the learning media has a strategic role in providing great visual experience to the students. One of the relevant learning media that can be used to help students in studying calculus is the geogebra program.

The contents of Calculus are limit, differential, and integral. Differential contents are: (1) using derivatives to solve optimization and change problems; (2) using first and second derivatives to determine gradients, extremes, turning points of polynomial and rational functions. In the case of real-life situations involving polynomial changes, the involved mathematical model is usually associated with the concept of a polynomial function, including its algebraic formulation, along with its tabular and graphical representations. However, by using geogebra, students can easily get an algebraic expression of a polynomial function; the curve equation appears as an independent object. Therefore, the process of making and applying mathematical models can change significantly.

By the using of computer for student access to the representation of interrelated polynomial variations, it is important to investigate how the students' approach to the polynomial model is influenced by geogebra's ability. This research focuses on fourth semester students of Mathematics Education Department of State Islamic Institute of Jember, Indonesia who are currently taking a course in mathematics learning media that develops mathematical and computer model with geogebra. This research aims to investigate how they formulate and apply polynomial model formed by technology tool.

Type of thinkers is divided by their preference of using solution method; visual and non-visual. There are three types of thinkers i.e. visualizer who is mostly use visual methods, nonvisualizer who is mostly using non-visual, and harmonic thinker who is using both methods of visual and non-visual [7]. Krutetskii defines two types of harmonic thinkers, namely abstract harmonics and pictorial-harmonic. Abstract harmonics are those that develop verbal-logical and visual-spatial components in equilibrium but have the tendency to use mental operations without the use of imaging tools. On the other hand pictorial harmonic are those who have a balance between the two but have the tendency to use mental operations by using visual image schemes.

The degree of visuality is the extent of how the subjects use the visual solution process to solve a given problem. The student's visuality score is based on the number of visual solution processes in the written test. The student studied the relationship between spatial ability, problem-solving performance and the level of fifth grade student's visuality. Moses found that the performance and level of visuality were correlated with spatial abilities and problem solving [8]. However, other several researchers assessed Moses's research as having many limitations. To overcome these limitations, Suwarsono developed an instrument named The Mathematical Processing Instrument (MPI). This instrument includes two parts and offers a visual and non-visual solution that is possible for the given problem. The first part includes 30 mathematical questions, and the second contains the written descriptions of different methods commonly used by students who are trying to solve story problems in Part I [9].

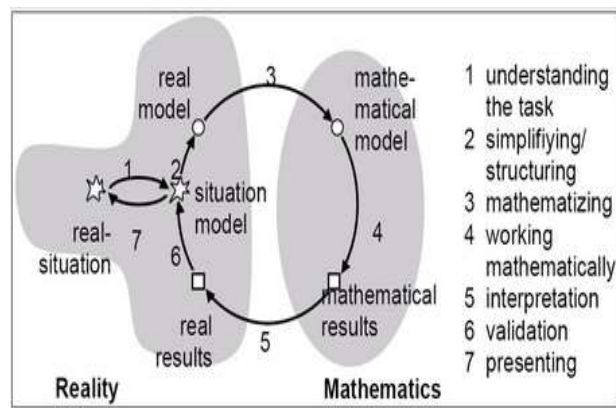
Students of Mathematics Education Department of State Islamic Institute of Jember also have different abilities and degrees of visuality in solving math problems. There are students who prefer to use visual methods and some prefer non-visual methods, and there is a mixture between the two, therefore the selection of research subjects is based on their degree of visuality.

## 2. Literature Review

Modeling is the link between mathematics and the real world. In other words, modeling can be a process which can support students in gaining understanding of mathematics through real-world contexts. Conversely, mathematics can be a tool to help students understanding real life. More specifically, mathematical modeling can be interpreted as the use of mathematics to solve problems

outside the mathematical context in mathematics way [10]. In mathematical modeling, context or real problem is arranged in a mathematical model to make the problem-solving process easier. The process of forming the mathematical model is through abstraction and idealization based on mathematical principles.

Based on the project of Didaktische Interventionsformen für einen selbständigkeitsorientierten aufgabengesteuerten Unterricht am Beispiel Mathematik (DISUM), the modeling process involves seven steps [11] as illustrated in Figure 1. The first stage is construction, which is to understand the known situation of the problem. At this stage students must construct the situation model. The second stage is simplifying/structuring, means structuralizing the situation. For example, by using variables to simplifying the situation. The third stage is mathematization, which is to transform the real model into a mathematical model, for example by creating an equation that contains a variable. The fourth stage is working mathematically, like counting, etc. to get mathematical results. The fifth stage is interpreting, that is, to interpret the mathematical results into the real world. The next stage is validation, means validating the results. The last stage is exposing to reveal the results obtained.



**Figure 1.** Modeling process.

From the description of the modeling process, it appears that modeling activities involve steps that enable students to develop various mathematical abilities. When students perform modeling tasks, especially when they try to understand the problem, they formulate questions about the context and express their thoughts in various representations, such as words, diagrams, worksheets, equations and graphs. Students are also encouraged to connect their mathematical knowledge to the context at hand. They will think about the use of their mathematical knowledge and try to find its connections with the problem. This activity allows students to develop their ability to recognize the role of mathematics in the real world. According to [12] in the process of mathematical modeling, the activity of simplifying or structuring, involves several activities of analyzing elements of a problem, identifying important features, creating assumptions that might aid analysis, identifying sub-problems and essential components, expanding components and finding suitable representations to clarify, exploring selected components, working mathematically, and finally defining the ways in which problems can be solved. That process involves a long chain of reasoning

In mathematical activity, students also have to do the reasoning mathematically, using concepts, procedures, facts and mathematical tools to find the appropriate model, so that problem can be solved easily. Furthermore, in mathematical work activities, students are encouraged to use mathematical tools to resolve the context at hand. To interpreting and validating the results phase, students are required to evaluate and reflect on their results. Through this activity, students can also review and develop more general models and solutions. Furthermore, in the last step, students are accustomed to being able to express or communicate the results correctly. This will encourage students to make reliable arguments and rational decisions.

The emphasis given to the particular goals behind mathematical modeling in education has enabled researchers to distinguish between different perspectives [13]. Thus, both conceptual modeling and contextual modeling perspectives are important roots to place the theoretical stance of this research. Realistic situations are seen as providing opportunities to gain students' understanding of broad polynomial functions, such as the rate of change and the constant parameters involved in the polynomial graph. Following that initial baseline, the contextual problems are seen as opportunities for conceptual development, linking various aspects of polynomial models, such as linking tables to graphs and equations in finding answers to specific problem-based questions. Different theoretical perspectives have shown the possible way to see the effect of introducing digital tools in students' understanding of mathematical models. One way to address the interaction between modeling and technology is by focusing primarily on the medium used by modelers and stands on the idea of collective action, which recognizes the interactive influence between users and technological tools [14]. From that point of view, the use of multi-representational technology tools determines the representational choices of students [15] and this is reflected in the modeling approach when completing modeling tasks. Research on strategies developed by students in problem-solving tasks has shown that they tend to avoid algebraic work in solving problems and prefer non-algebraic routes such as those based on arithmetic reasoning, trial and error, backward work, etc. Those are sum up to certain compulsions to calculate rather than to do algebraic work [16].

A similar movement to delay algebraic approach is reported by Yerushalmy, where students use technology intensively in the algebra school's function approach, through modeling tasks. Based on the proposed linear function task for student's pair in three separate interviews, the study showed how the students' strategic approach to the problem evolves: from graphical representation to numerical methods, from relationships between quantities to graph, ultimately to the algebraic expressions [17].

As stated earlier, technology brings more opportunities for students to determine which representational mode is most efficient for formulating and applying mathematical models to real situations. Therefore, students' approach to mathematical modeling tends to developing in such versatile contexts and adjusting to different levels of mathematization (from the real world to mathematics or the mathematical world). In a developing process such as representational decisions, joint actions between the student and the tool become important concepts. The tool can offers answers (e.g. algebraic expressions) as a result of representational alternate actions of the user (e.g. chart plotting). This study looks at ways in which students approach the polynomial model in a multi-representational environment, more precisely to see how they formulated and applied in a series of tasks.

### 3. Methodology

This study used a qualitative approach and the type of research conducted was descriptive. The subjects of this study were six students of 4th semester of Mathematics Education Department, State Islamic Institute of Jember, Indonesia academic year 2017-2018. All subjects had different degree of visuality, i.e. two visual students, two harmonic students, and two non-visual students.

**Table 1.** List of research subjects based on visuality level.

No	Code	Suwarsono MPI Score	Degree of visuality
1	V01	48	Visualizer
2	V02	50	Visualizer
3	H01	28	Harmonic thinker
4	H02	26	Harmonic thinker
5	N01	16	Nonvisualizer
6	N02	10	Nonvisualizer

The study was conducted in computer lab where students were organized in pairs. Each pair had one computer to work on. In each task, students were to find a mathematical model for problem situations by selecting information, interpreting the situation, creating a model and applying it to find answers to specific questions about the real context.

Some of the tasks required the students engaged in real data collection. All tasks were meant to enable the use of mathematics, namely the concept of polynomial variation and polynomial function, in relation to contextual questions. Tasks evolved from problem that aimed at generating common polynomial models with more focused issues where the context was explored to stimulate the use of polynomial models to obtain specific solutions.

Students as subjects already have some experiences in using geogebra because they were taking mathematics learning media subject. They are already familiar with Graphics View, Algebra View and Spreadsheet View from said software. Students are not fully aware of the fact that once they create geometric constructions in Graphic Display, the equations are shown in Algebra Views, because of the interactive nature of graphics and algebra in geogebra. The researcher decided to allow students to find their own additional details of the program while performing the task. Students were not forced to use computers but they were free to choose how to complete each task. Therefore, in the same task, some students can work with geogebra, others only with paper and pencil, and even others can use both. Data were collected using questionnaires, tests, observations, and interviews. The questionnaire is a Mathematical Processing Instrument (MPI), which is used to obtain data about the student's degree of visuality. The test in this study is the MPI test. The test aimed to obtain data about the student's degree of visuality. The questionnaire and the MPI test were adopted from MPI Suwarsono consisting of 30 mathematical problems [9]. The interview is conducted to obtain data in the form of students oral expression about modeling activities in solving the problem of polynomial function.

Data analysis used in this research consisted of two types. First, the student's degree of visuality analysis was determined by summing the student's MPI score. According to Suwarsono for any mathematical problem, score 2 is given if the student uses a visual solution method, a score of 1 is given if the student does not show any visual or non-visual methods, and a score of 0 is given if the student uses a non-visual solution method. Therefore, the value of student visuality ranges between 0 and 60 [9]. After that, the results of the student MPI test then be classified based on three levels of visualization i.e. visualizers, nonvisualizers, and harmonics. Second, the data analysis of student modeling in solving polynomial function problem applied Miles & Huberman model. Activities in this data analysis are data reduction, data display, and conclusion drawing or verification [18]. The categorizing process of the level of student visuality can be done with specific requirements such as table 2.

**Table 2.** Criteria for student visuality rate.

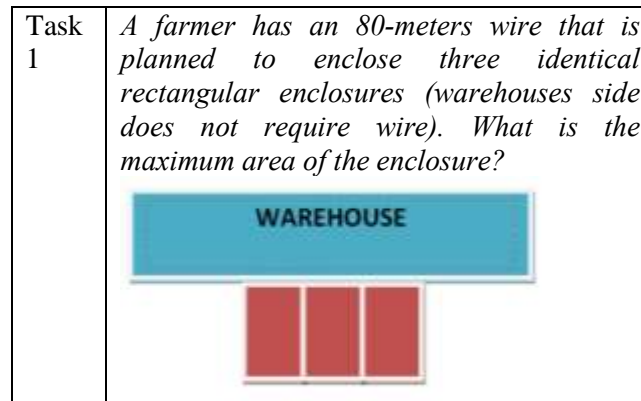
MPI Suwarsono Score	Degree of Visuality
0 - 20	Nonvisualizer
21 - 40	Harmonic thinker
41 - 60	Visualizer

Source: [7, 19, 9]

#### 4. Results and Discussion

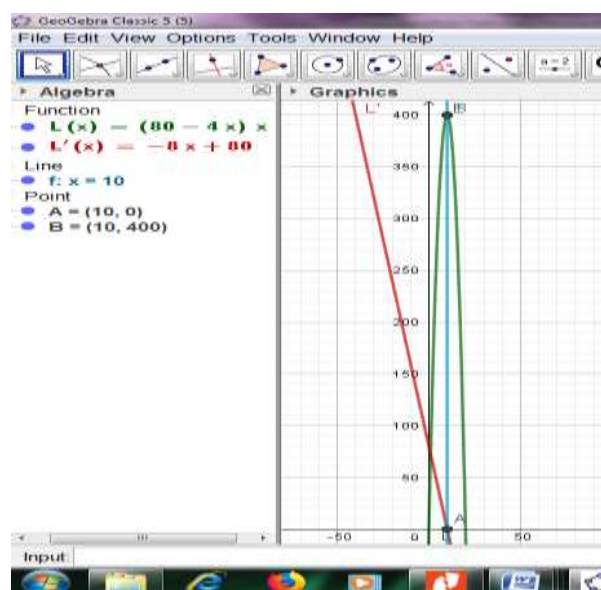
Three tasks related to the problem of polynomial function have been given to the research subject to be completed using the geogebra program; they are free to choose using geometrical approach or algebraic approach to complete the task. The results showed that visual students prefer to use geometric approaches and avoid algebraic work because they preferred visualization of real problems into graphic form, whereas harmonic and non-visual students preferred the algebraic approach and avoided geometric work because they found it easier, simple and not complicated.

The following is a visual, harmonic, and non-visual student's modeling descriptions in solving three polynomial function problems using geogebra approach.



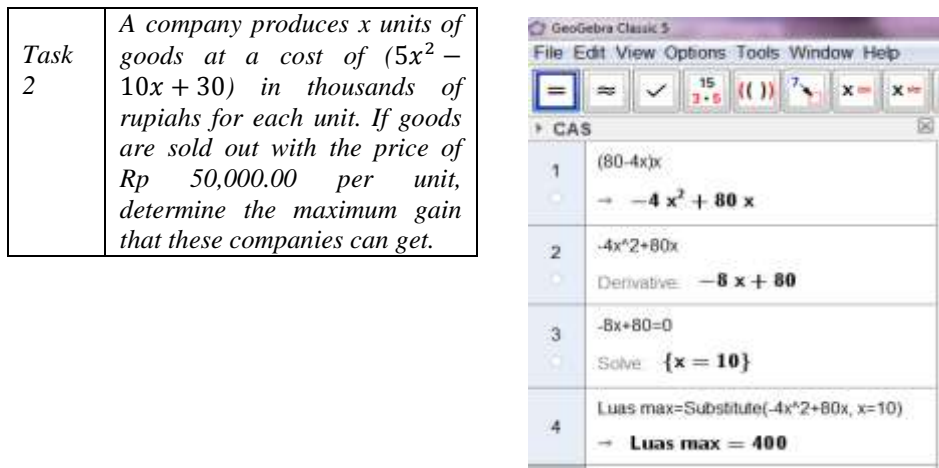
**Figure 2.** Task 1.

Visual students completed task 1 by using geometrical approach. That was by creating model from the real problem. They modelled the width of the enclosure as  $x$  and the length of enclosure as  $y$ . From those assumption, they generated an equation of the used long-wire to enclose the enclosure as  $y + 4x$  (the total ware house side) and that must be the same as 80 (total wire), then it represented as  $y + 4x = 80$ . Afterall, they changed the equation to  $y = 80 - 4x$ . Using the reactangle area formula, they generated new equation of the enclosure area as  $L = (80 - 4x)x$  and enter the equation in the input bar and press enter. Then the graphics view displayed a curve in the form of parabola and the algebra view also displayed the quadratic function  $L(x) = (80 - 4x)x$ . To find the maximum area of the enclosure, they looked for the first derivative of  $L$  first by typing  $L'$  on the input bar and press enter, the algebra view then displayed the function  $L(x) = -8x + 8$  and the graph appeared as a straight line. After that, they looked for the intersection between the  $X$ -axis and the straight line using the contraction tool "intersect two objects" on the geogebra, and resulting point  $A(10, 0)$ . Next the visual students decided to include the equation  $x = 10$  on the input bar and obtain a vertical line that intersects the parabola at point  $B(10,400)$ . After that they found the maximum enclosure area, and it was  $400 \text{ m}^2$ .



**Figure 3.** The solution of visual student in solving task 1.

On the other side, harmonic and non-visual students preferred the algebraic approach in completing task1. They used the Computer Algebra System (CAS) facility by creating a mathematical model in the form of a quadratic function  $(80 - 4x)x$  of the given real problem. Then they looked for the first derivative of the function by pressing the "derivative" icon to obtain linear function  $-8x + 80$ . After which, they solve the linear equation by clicking the "solve" icon so that it is solved  $x = 10$ . They found the maximum area by substituting  $x = 10$  into quadratic equations  $-4x^2 + 80x$  as shown in figure 2 below. Although the way is different, they got the same end result.

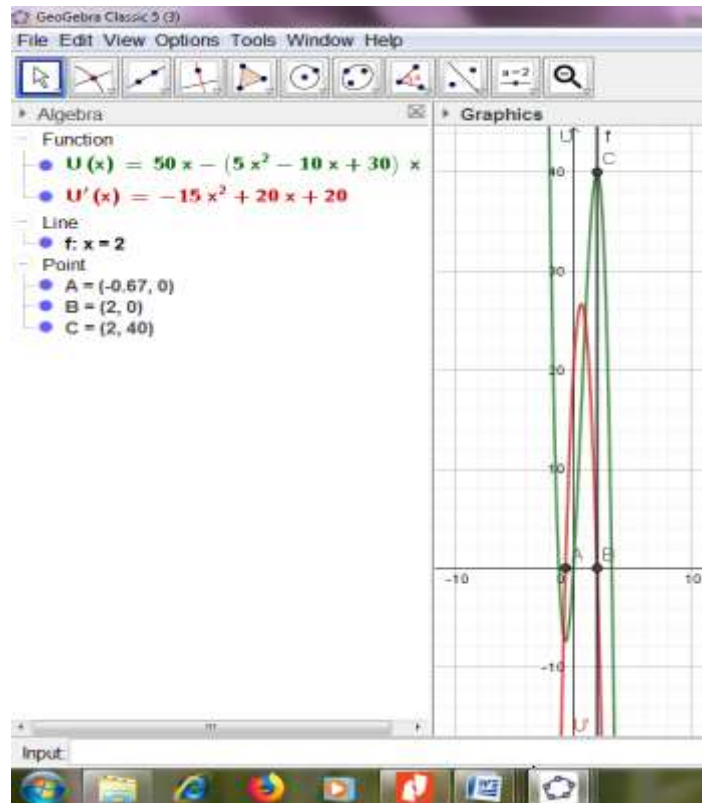


**Figure 4.** The solution of nonvisual and harmonic student in solving task 1.

The visual student completed task 2 using geometrical approach i.e. by writing production cost  $x$  unit as  $(5x^2 - 10x + 30)x$  and sales cost  $x$  unit as  $50x$ . Both costs are in thousands of rupiah. By using the formula Profit = Cost of sales - production costs, they generated a mathematical model  $U(x) = 50x - (5x^2 - 10x + 30)x$  and enter the function in the input bar and then press enter, then the graphic view appeared curves in the form of polynomial curves and the algebra view also appeared the polynomial function.

To find the maximum profit for the company, they looked for the first derivative of  $U(x)$  first by typing  $U'$  on the input bar and press enter, then the algebra view showed the function  $U'(x) = -15x^2 + 20x + 20$  and the resulting graph is a parabola opens downward. After that they looked for the point of intersection between X-axis and parabola using construction tool "intersect two objects" on geogebra so that it could get point A(-0.67,0) and point B(2,0). Next the visual student decided to include the equation  $x = 2$  on the input bar and obtain a vertical line, using the construction tool "intersect two objects" on the geogebra again they looked for the intersection between the lines  $x = 2$  and the polynomial curve and obtained point C(2,40). Thus, they found the maximum profit the company earned, and that was Rp. 40,000.00.

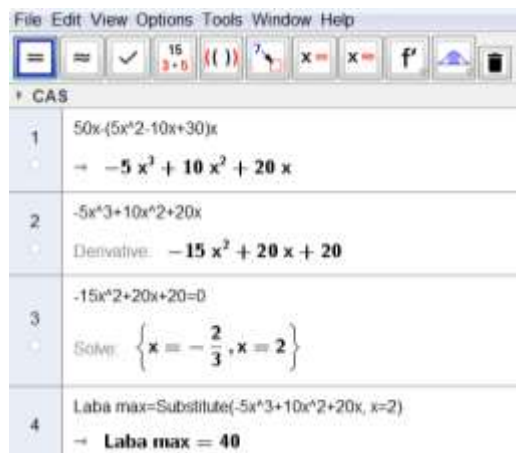




**Figure 5.** The solution of visual student in solving task 2.

Harmonic type and non-visual type of students avoided geometry work, they preferred the algebraic approach in completing task 2 by using the Computer Algebra System (CAS) facility. They use the same way with visual students, which is creating the mathematical model in the form of polynomial functions  $50x - (5x^2 - 10x + 30)x$  from the real problem, then finding the first derivative of the function by using the "derivative" icon to obtain quadratic function  $-15x^2 + 20x + 20$ . After which they solve the quadratic equation by using the "solve" icon so that the result is  $x = 2$  or  $x = -\frac{2}{3}$ . They pick  $x = 2$  to be substituted into a polynomial function  $-5x^3 + 10x^2 + 20x$  as shown in figure 4. Thus, they found the maximum profit that the company can earn was equal to Rp. 40,000.00. Although the approach used is different, they could get the same result as the visual students.

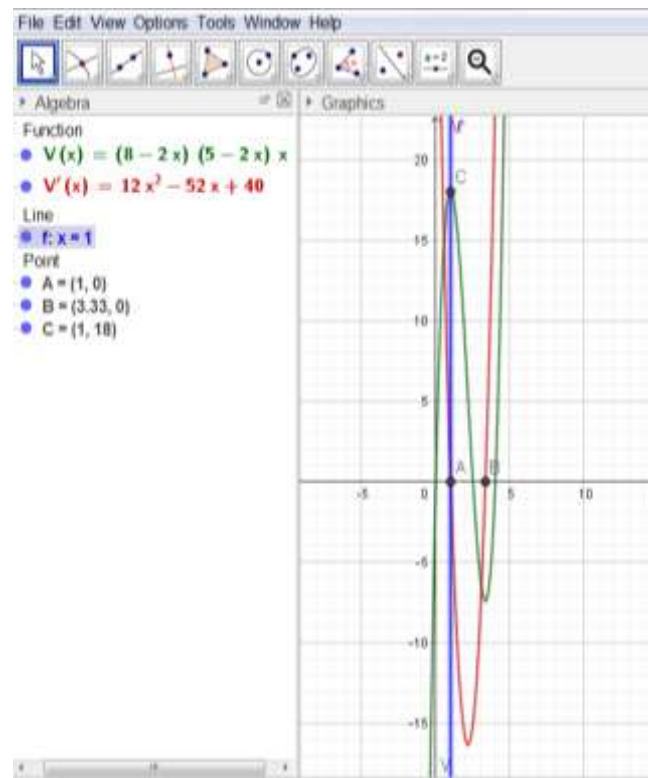
**Task 3** A rectangular cardboard with a width of 5 dm and a length of 8 dm will be made without a lid. Each corners of the cardboard then cut with the shape of square and the size of  $x$  dm. What is the square size (length, width, height) so that the volume is maximum?



**Figure 6.** The solution of nonvisual and harmonic student in solving task 2.

The visual student completed task 3 by using geogebra geometrical approach, they assumed the box height =  $x$ , box width =  $(5 - 2x)$  and box length =  $(8 - 2x)$ . By using the volume's formula they then created a mathematical model for the box volume  $V = (8 - 2x)(5 - 2x)x$  then entered the equation in the input bar and then pressed enter, the graphic view will display curves in the form of polynomial curves and the algebra view will also displaying polynomial function  $V(x) = (8 - 2x)(5 - 2x)x$ .

To find the maximum volume, first they searched for the derivation of  $V$  by typing  $V'$  on the input bar and then pressed enter, the algebra view will show the  $V'(x) = 12x^2 - 52x + 40$  graph which is the shape of a facing up parabola. After that, they looked for the parabola's intersection point with the X axis by using construction tool "intersect two objects", they then obtained two points, namely point A(1,0) and B(3.33,0). Next the visual student decided to enter the equation  $x = 1$  on the input bar and drew a vertical line that intersects the polynomial curve at point C(1,18). Thus they found the maximum volume of the box, and that was  $18 \text{ dm}^3$ . They understand that the maximum volume occurs when the value  $x = 1$



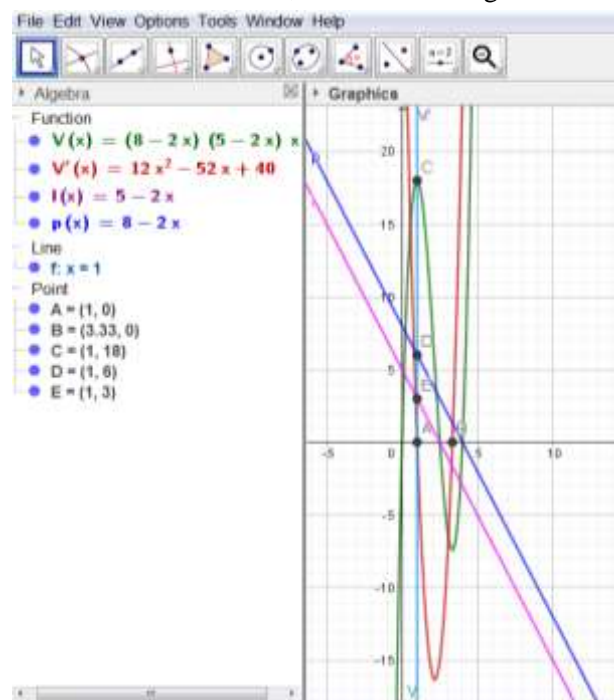
**Figure 7.** The solution of visual student in solving task 3.

However, the question is not how much the maximum volume is but the size of the box so that the volume will be maximum. Here the students think hard about how to find the length, width and height of the box so that volume will be maximum by using geometrical approach. As they couldn't find it, finally they opened the CAS facility to calculate the length, width and height by substituting values  $x = 1$  into equation length =  $8 - 2x$ , width =  $5 - 2x$ , and height =  $x$  as shown in figure 6 to obtain box length = 6 dm, box width = 3 dm, and box height = 1 dm.



**Figure 8.** The final solution of visual student in solving task 3.

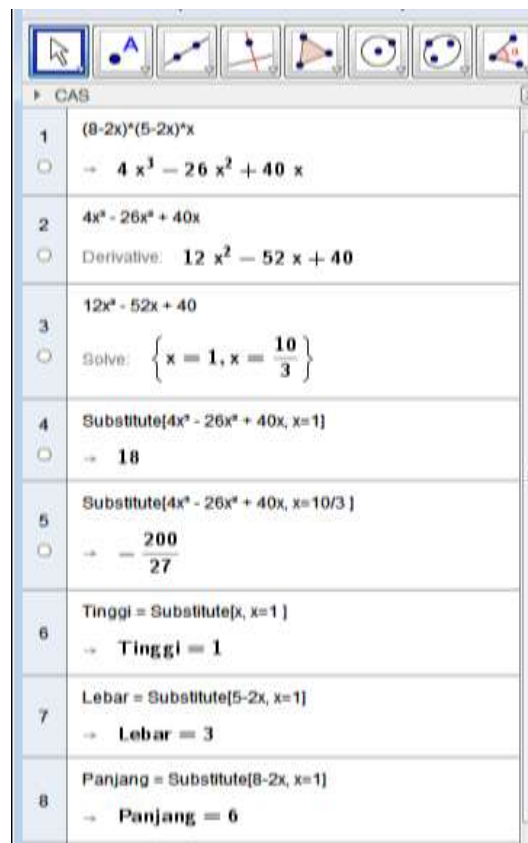
Apparently visual students were not satisfied with the results they obtained, they decided to try again using geometrical approach as shown in figure 7. After finding the maximum volume at  $x = 1$  they enter the equation  $p = 8 - 2x$  and  $l = 5 - 2x$  on the input bar and obtained two straight lines in the graphic view. To find the length of the box they looked for the intersection between the lines  $x = 1$  and  $p = 8 - 2x$  so the result is point D(1,6) which means box length is 6 dm. Whereas, to find box width, they looked for intersection point between line  $x = 1$  and  $l = 5 - 2x$ . Finally, they found point E(1,3) which means that the box width was 3 dm, and the box height was 1 dm.



**Figure 9.** The second solution of visual student in solving task 3.

Meanwhile, non-visual and harmonic students preferred the algebra approach in completing task 3 by using the Computer Algebra System (CAS) facility. They solved it by using the same way with visual students, which was by making a mathematical model in the form of polynomial function  $(8 - 2x)(5 - 2x)x$  of the real problem given. Then they looked for the first derivative of the function

so that the quadratic function is obtained  $12x^2 - 52x + 40$ . After that, they solved the quadratic equation so they obtained  $x = 1$  or  $x = 10/3$ . After substituting each value of  $x$  into the polynomial function and obtaining the maximum volume, they chose  $x = 1$  to be substituted into linear functions *height* =  $x$ , *width* =  $5 - 2x$  and *length* =  $8 - 2x$  as shown in figure 4, finally they found that the length = 6 dm, width = 3 dm, height = 1 dm. Although the approach that they used was different, they got a same final result.



**Figure 10.** The solution of nonvisual and harmonic student in solving task 3.

The results of this study indicate that visual students use graphical representation of mathematical models as the main source of their model analysis, which allows them, to avoid solving equations. Student actions (plotting graphs) followed by a prompt from the tool (equation). This in turn generates new actions from students: entering a formula to get vertical or horizontal line. The tool then provides the point of intersection and its coordinates; the students look at the values shown to find solutions to the equations and thus apply the models to find their answers. The way computers are used describes the repetitive process of joint action between students and tools. Moreover, there is also action and reaction between computer models, the mathematical world and the real context in giving meaning to variables and to algebraic expressions.

Visual student's solutions showed that, in many situations, students could obtain the solution algebraically. Therefore, it seems that working with algebraic expressions and solving polynomial equations is not difficult for most students. The choice for geometric manipulation of the polynomial model is to support prior research that highlights students' preferences for non-algebraic approaches [16,20]. In this study, such preferences can also be explained for reasons that go beyond the potential difficulties in algebraic manipulation. The ability of computing tools is assimilated by students and reflected in the use of geometric objects (lines, intersections, dots, coordinates) to find alternative approaches to explore the model.

Geometrical representation becomes their reference object in the modeling process and a means for obtaining algebraic equations in Algebra Views. This seems to be a great example of the "algebracising" mode of software used [3]. In fact, students were initially surprised by this utility provided by the software and quickly began to fit their model explorations. Likewise, they realize that by entering an equation in the input bar ( $x = k$  or  $y = mx + c$ ), the geometric object immediately appears in Graphics View. As a result, direct translation from geometry to algebra and from algebra to geometry has an influence on student modeling approaches. Regarding the application of the model, the strategy used is essentially geometric, taking advantage of the possibility of inserting newlines and connecting objects in Algebra and Graphic Display. This is the main reason why many of the equations involved in the problem were solved from a geometrical point of view, by cutting the line and obtaining the coordinate of the intersection.

Meanwhile, Non-visual and Harmonic students preferred using algebraic representation by selecting CAS facilities. They preferred to use procedural steps such as when solving problems using paper & pencil. They were not interested in graphical visualization that they found more difficult. That was because it required a deep understanding of having to relate the real situation with the graphs and algebraic equations that follow. The CAS function is almost the same as the calculator, which instantly answers the commands entered, so they do not have to think conceptually because it has been answered directly by the tools used.

That result is slightly different from previous Farihah research [21]. Farihah, in previously research, said that harmonic and non-visual students preferred to use geometric approaches rather than algebraic approaches. They tended to avoid the usage of algebraic approach when they were asked to solve a problem related to build a mathematical model in linear function.

The data revealed how students were guided by and simultaneously guided computing tools to explore and understand polynomial models, demonstrating joint action between students and media [14], modeling activities in the "technological world" [22].

## 5. Conclusion

Based on the results of the questionnaires, tests, observations, and interviews conducted on students, the researchers conclude that students' modeling in solving the polynomial functions problems using geogebra approach is varied. Visual students prefer geometrical approach and avoid algebraic work. They prefer to visualize the real problem into graph. Non-visual students and harmonic students precisely avoid geometry work and prefer algebraic approaches. They find it easier to work with CAS than to construct charts. Geometrical approach is perceived by students to be more difficult because it requires a deep conceptual understanding. The most interesting aspect of polynomial function modeling using geogebra is that it appears to provide students with powerful tools for researching, reinventing the results for themselves. It helps them to increase the depth of their understanding beyond the traditional approach, where the concept and the results are presented as facts and rules to be learned. The gap between intuition and theory is very broad in calculus and in geogebra. Students have a good tool to bridge this gap. Recommendation for further researcher is to examine students' approaches to the contextualized problems in the "technology world" of the modeling activity using more students as the subjects of the research.

## Acknowledgement

The author is thankful to the anonymous referee for valuable comments and kind suggestions.

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