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The synergy of students' use of paper-pencil techniques and geogebra in solving analytical geometry problems

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Abstract. This study aimed to analyze the synergy relationship between students' strategies in solving geometry analytic problem by using paper-and-pencil and geogebra. This study used descriptive and qualitative approach. The subjects of this study were six of the fourth semester students of Mathematics Education Department, State Islamic Institute of Jember, East Java Indonesia academic year 2017-2018 with different degree of visuality; two visual students, two harmonic students, and two non visual students. Based on the result of research analysis, it was shown that there were different acquisition degree of geometrical abilities concerning the students' processes of instrumentation and instrumentalization when they work together in a geogebra and paper-and-pencil media.

1. Introduction

Geometry is a branch of mathematics. James stated that geometry is a science that deals with the shape and size of objects [1]. Analytical geometry is the basic course of geometry that studies two-dimensional object. This course aims to develop the ability of students to understand the geometry equation in 2D plane in the form of vector equations, canonic, and parameters, the position of the line against other lines, and the position of the line against the cone slices. Until now geometry is still a difficult subject for students, this can be seen from the low students score of analytical geometry, transformation geometry, and geometry systems [2]. Therefore, teachers need to integrate technology in learning.

Integration of computational technology in mathematics learning especially in the use of Dynamic Geometry Software in the context of student understanding of analytical geometry through problem solving is needed to improve student understanding. This study focuses on interpreting students activities when they solve geometry problem analytically by analyzing the relationships and synergies between geogebra and paper-pencil, and geometrical thinking [3]. This research focuses on the use of geogebra as a free Dynamic Geometry Software which facilitate students several basic features of Computer Algebra System (CAS). Geogebra software connects synthetic geometric constructs (geometric windows) to analytic equations, coordinates representations and graphs (algebra windows) [4].

This study aimed to analyze the relationship between student problem solving strategies by using paper and pencil based and geogebra (ICT based). Laborde stated that tasks completed by using



Dynamic Geometry Software may require different strategies than those task when solved with paper-pencil [5]. This fact shows the influence of feedback given to students. By analyzing and comparing the settlement process in both environments, taking into account interactions (students and geogebra students) the main objectives of this study are to: 1) analyze the process of instrumentation and instrumentalization of students by considering the level of acquisition of geometric abilities from this process; and 2) explore the effect on resolution strategies when using paper-pencils and geogebra.

A theory which is usefull to understand the obstacles of the use of technology, in this study is using geogebra, is the perspective of instrumentation [3]. The instrumental approach used has been implemented to the research of Computer Algebra System (CAS) into understand several mathematics concept and also to Dynamic Geometry Systems. The instrumental approach distinguishes between instrument and artifact. Rabardel and Vérillon claimed the significan on emphasizing the difference between the instrument and the artifact [6]. A technical system or a machine does not immediately constitute a tool for the subject; it then becomes an instrument when subject has been able to fit it for her/himself. This transformation process of a tool into a useful instrument then called as instrumental genesis. During the instrumental genesis, students built mental schemes. In these mental schemes, conceptual and technical components are coming to substitute each other [7]. This process depends on the characteristic of the artifact and also complex, its constraints and affordances, and also on the knowledge of the user. The process of instrumental genesis has two dimensions, the instrumentation and the instrumentalization.

Instrumentation is a process through which the constraints and the affordances of the tool affect students' way in solving problem and the corresponding emergent conceptions [3]. This process then is used during the emergence and evolution of students' schemes when they solve the problem [8]. Instrumentalization is a process which is happened when the students use their knowledge to guide the way the tool is used and in a sense shapes the tool [3]. This process enriched the artifact, or to its impoverishment [8].

The degree of acquisition of geometric capabilities regarding this process in the context of the given task is defined as in tables 1 and 2 below.

Table 1. Instrumentation degrees of acquisition of geometrical abilities

Degree of Acquisition	Indicator
High	Students understand the geogebra affordances (such as constructing and using new tools) and they don't fine some obstacles.
Medium	They know and are able to use geogebra to solve geometry problem.
Low	They fine several obstacles and use few tools to solve the problem. This leads to an impoverishment of the proposed tasks.

Table 2. Instrumentalization degrees of acquisition of geometrical abilities

Degree of Acquisition	Indicator
High	Students realize the use of geometric and algebraic windows and use their geometrical understanding to solve the problem.
Medium	Students are able to coordinate the application of both algebraic and geometric windows. They construct it based on the geometry properties and draw it.
Low	Students concentrate only in measuring equipment rather than considering geometric properties in that construction.

The typologies of behaviors that arise from the interpretation of the degrees of acquisition of the instrumentation and instrumentalization processes are defined as in table 3.

Table 3.Typologies of behaviors of the degree of acquisition

Typologies of behavior	Indicator
<i>Autonomous</i>	The instrumentalization and instrumentation degrees of acquisition are commonly high
<i>Instrumental</i>	The instrumentation and instrumentalization acquisition degrees acquisition are medium to high
<i>Procedural</i>	The instrumentalization degree is lower than instrumentation degree
<i>Naif</i>	Both instrumentalization and instrumentation degrees of acquisition are low

First thing first, the researcher chose a different problem to be solved first by using paper and pencil, and secondly the researcher used geogebra. To analyze connectivity and synergy between student resolution strategies in both environments, the problem chosen is a similar problem so can know the strategies they use to solve the problem are different or the same.

Researchers considered the differences in the use of drag tools in the problem solving process when analyzing the students product of geogebra [9].It is important to distinguish the main functions of dragging in the problem solving process. For example, the purpose of a drag test is to check the correctness of construction. This can be considered a validation criterion for geometric construction. The researcher also determined a number of terms that would be used in geogebra's student resolution research such as figure and drawing. The researcher uses these terms with their meanings commonly used in the context of Dynamic Geometry Software [10].

Holle brands used the different of drawing and figure to describe how students understand the generated representations on the computer screen. For instance, when student draw a rectangle only consider about measurement equipment, the figure does not pass the dragging test and it is considered a drawing. Therefore, to draw a figure with its geometrical properties, student has to know the geometrical properties of the object and certain equipment(technical knowledge of the software)[11].

Inatechnology environment, students are able to develop alternative strategies and explore different strategies that could not be easily explored in a PPB environment [12]. Moreover, Iranzo-Domenech stressed that when students solve problems using technology, they tend to develop different competencies based on their mathematical knowledge [13]. Iranzo and Fortuny's research report states that the existence of different acquisition degree of geometrical abilities concerning the students' processes of instrumentation and instrumentalization when they work together in a geogebra and paper-and-pencil media [14].

2. Method

This study uses a qualitative approach and the type of research conducted is descriptive. The subjects of this study are six of 4th semester students of mathematics education, education and teacher training faculty, State Islamic Institute of Jember, Indonesia academic year 2017-2018, with different visuality level, ie two visual students, two harmonic students, and two non-visual students. Researcher used table 4 to chategorize students visuality level based on Suwarsono Mathematical Processing Instrument (MPI) score, which is:

Table 4. Criteria of Students Visuality Level	
Suwarsono MPI Score	Level of Visuality
0 – 20	<i>nonvisualizer</i>
21 – 40	<i>harmonic</i>
41 – 60	<i>visualizer</i>

The students were those who took mathematics learning media courses and able to use the geogebra program, and had one of the types of visuality levels mentioned above.

Table 5. List of Research Subjects Based on Visuality Level

Code	Suwarsono MPI Score	Level of Visuality
V01	48	<i>Visualizer</i>
V02	50	<i>Visualizer</i>
H01	28	<i>Harmonic</i>
H02	26	<i>Harmonic</i>
N01	16	<i>Nonvisualizer</i>
N02	10	<i>Nonvisualizer</i>

This research was conducted in a computer lab and students were organized in pairs, each student had one computer to do the assignment. Students given two similar assignments related to the problem of analytic geometry fields, the first, students asked to complete using a paper-pencil, then the second, they asked to complete using geogebra help.

The method of data collection is done by using questionnaires, tests, observations, and interviews. The questionnaire form is a Mathematical Processing Instrument (MPI), which used to obtain data about the student's level of visibility. The test used in this study is the MPI test, it aimed to obtain data about the student's level of visibility. The MPI questionnaire and test were adopted from MPI Suwarsono which consisted of 30 mathematical problems [15]. Whereas interviews were conducted to obtain data in the oral expressions about students' thinking activities and the strategy use of student pencil and geogebra media in solving problems in plane analytical geometry.

Analysis of the data used in this study consisted of two types. First, the student's level of student visuality analysis is determined by summing the student's MPI score. According to Suwarsono, for each math problem, a score of 2 is given if students use the visual solution method, a score of 1 is given if the student does not indicate the existence of a visual or nonvisual method, and a score of 0 is given if the student uses a non visual solution method [15]. Therefore, the students' visuality values range between 0 and 60. The results of the student MPI test then classified based on three levels of visualization ie visualizers, non-visualizers, and harmonics. Second, data analysis synergized the use of paper-pencil and geogebra techniques in solving problems in analytic geometric fields using the Miles & Huberman model. Activities in this data analysis are data reduction, data display, and conclusion drawing or verification [16].

3. Research Result

Two tasks related to plane analytical geometry problems were given to the research subjects to be completed using two techniques, paper-pencil and geogebra. The following is a description of the visual, harmonic, and non-visual student strategies in solving these two problems.

<i>Circle problem</i>	<i>Find the center and the radius of a circle such that $P = (1, -1)$, $Q = (3, 5)$ belong to the circle and the circle's center belongs to the line r of equation $r: x + y + 2 = 0$.</i>
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Visual and harmonic students solve circle problems use paper-pencil techniques through several stages. First, they determine the coordinates of the midpoint of the PQ line then looked for the equation for the PQ line which is $6x - 2y - 8 = 0$. After that they determine the PQ line gradient first then determine the new line gradient perpendicular to the PQ line, then look for the new line equation which is $s: x + 3y - 8 = 0$. Because both lines r and s passed through the center of the circle, the intersection of the two lines is the center of the circle, using the elimination method obtained by the coordinates of the intersection $x = -7$ and $y = 5$ so that the center of the circle is found $(-7, 5)$. Their final step is to find the distance of the center of the circle to point P which is the radius of the circle as can be seen in figure 1.

<p>Diketahui garis $r: x+y+2=0$ $P(1,-1)$ dan $Q(3,5)$ Koordinat titik tengah $PQ = T(2,2)$ Pers. garis PQ adalah: $\frac{y+1}{6} = \frac{x-1}{2}$ $6(x-1) = 2(y+1)$ $6x-6 = 2y+2$ $6x-2y = 8 \rightarrow m=3$ Garis s tegak lurus garis PQ maka pers. garis s adalah: $y-2 = -\frac{1}{3}(x-2)$ $3(y-2) = -(x-2)$ $3y-6 = -x+2$ $x+3y = 8$</p>	<p>Garis r dan garis s berpotongan di titik pusat lingkaran</p> <table style="width: 100%; border-collapse: collapse;"> <tbody> <tr> <td style="padding-right: 20px;">$x+y = -2$</td> <td>$x+y = -2$</td> </tr> <tr> <td style="padding-right: 20px;">$x+3y = 8$</td> <td>$x+5 = -2$</td> </tr> <tr> <td style="padding-right: 20px;">$-2y = -10$</td> <td>$x = -7$</td> </tr> <tr> <td style="padding-right: 20px;">$y = 5$</td> <td></td> </tr> </tbody> </table> <p>jadi titik pusat lingkaran = $A(-7,5)$</p> $AP = \sqrt{(-7-1)^2 + (5+1)^2}$ $= \sqrt{8^2 + 6^2}$ $= \sqrt{100} = 10$ <p>jadi jari-jari lingkaran = 10 sat</p>	$x+y = -2$	$x+y = -2$	$x+3y = 8$	$x+5 = -2$	$-2y = -10$	$x = -7$	$y = 5$	
$x+y = -2$	$x+y = -2$								
$x+3y = 8$	$x+5 = -2$								
$-2y = -10$	$x = -7$								
$y = 5$									

Figure 1. The Resolution Strategy of Visual and Harmonic Students in Solving Circle Problem with Paper-pencil

While Non-visual students solve this circle problem by determining the radius of the circle using the point to line the distance formula, but they could not use the formula so they abandoned this strategy because they got different values for the radius. The researcher concludes that the student strategy in Figure 2 refers to the system of equations: $radius = d(P, r)$ and $radius = d(Q, r)$

	$ PA = \frac{ pa + qb + r }{\sqrt{p^2 + q^2}}$ $= \frac{ 1 + (-1) + 2 }{\sqrt{1^2 + 1^2}}$ $= \frac{2}{\sqrt{2}}$ $ QA = \frac{ 3 + 5 + 2 }{\sqrt{2}}$ $= \frac{10}{\sqrt{2}}$
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Figure 2. The Resolution Strategy of Non-visual Students in Solving Circle Problem with Paper-pencil

However, when solving the problem of circles using the help of geogebra, they used a different strategy with paper-pencil. Each student group, both visual, harmonic and non-visual, has different strategy in solving circle problems. Visual and harmonic students have no difficulty in solving circle problems, they use geometric properties smoothly, but non-visual students experienced conceptual and technical difficulties.

In solving the problem, visual students followed several steps. First thing first, they entered the coordinates of the points P and Q, and the equation of the line f which is $f: x + y + 2 = 0$ in to the *input bar*. So that, the point P and Q as well as the line f were appeared at the geometry window. Second of all, they built any point A, which was located on the line f and then created segment \overline{PA} and \overline{QA} . Thirdly, they dragged point A along the line f until reached $|\overline{PA}| = |\overline{QA}|$ (The length of \overline{PA} and \overline{QA} is equal) such indicated in the algebra window. After finding the length of PA equal to QA which is 10 units, they concluded that A is the center of the circle and the length of the line PA or QA is the length of the radius of the circle (figure 3).

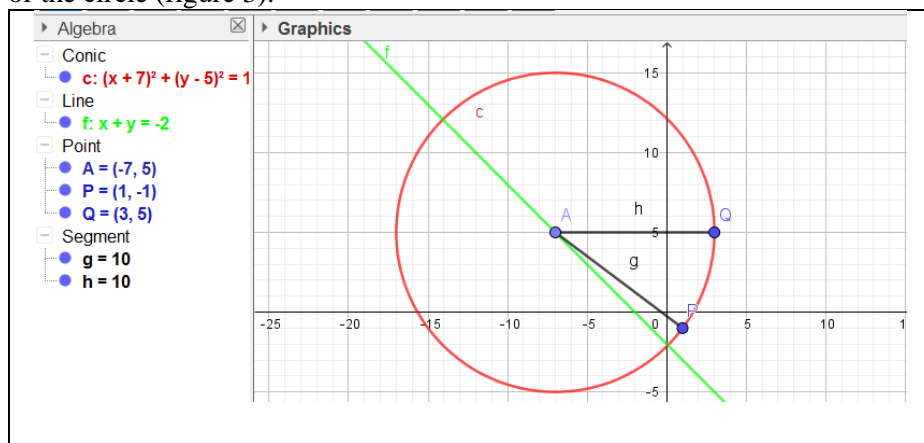


Figure 3. The Resolution Strategy of Visual Students in Solving Circle Problem with Geogebra

Meanwhile, non-visual and harmonic students solved the circle problem with the following steps: first, input the point P and point Q. Next input the equation $x + y + 2 = 0$ so that the output in geometry window was a picture of points P and Q and lines $f: x + y + 2 = 0$. Second of all, students used the circle tool through three points to construct the circle in the geometric window. Thirdly, they obtained the radius and coordinates of center point by interpreting the circle equation that appears in the algebra window: $(x - a)^2 + (y - b)^2 = R^2$. They found that the center circle coordinates of center circle is (-7.5) and the radius is 10 units (figure 4).

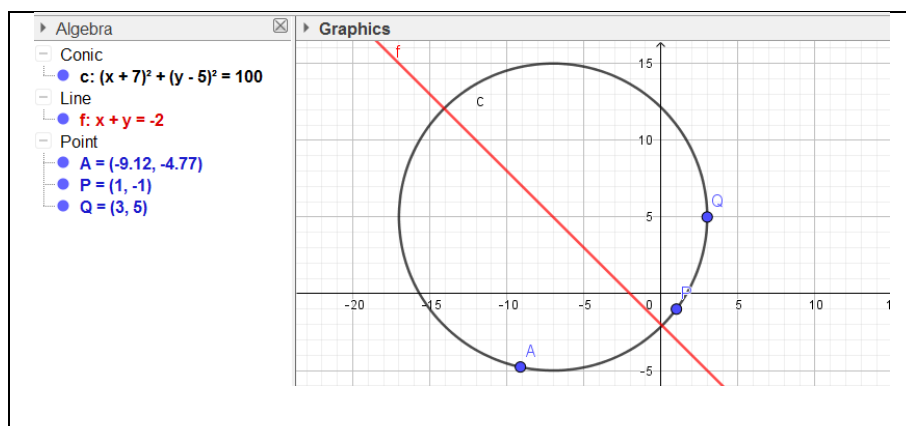


Figure 4. The Resolution Strategy of Harmonic and Non-visual Students in Solving Circle Problem with Geogebra

<i>Rhombus problem</i>	<i>A rhombus has two vertices $P = (-2, 1)$ and $Q = (0, -3)$ that form a diagonal of the rhombus. The perimeter is 20 cm. Find the remaining vertices and the area of the rhombus.</i>
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All research subjects solved the rhombus problem by using a paper-pencil technique first. When searching for one of the remaining vertices all research subjects both visual, harmonic and non-visual students used the formula of distance between two points so that the length of PA segment was equal to the length of QA segment. They had a little difficulty in finding vertices A coordinates but after they try, finally they got vertices A (3,1) coordinates. After that, they looked for node B in the same way, which is by trial and error until finally the vertices was obtained B (-5,-3). But non-visual students did not continue looking for the rhombus area.

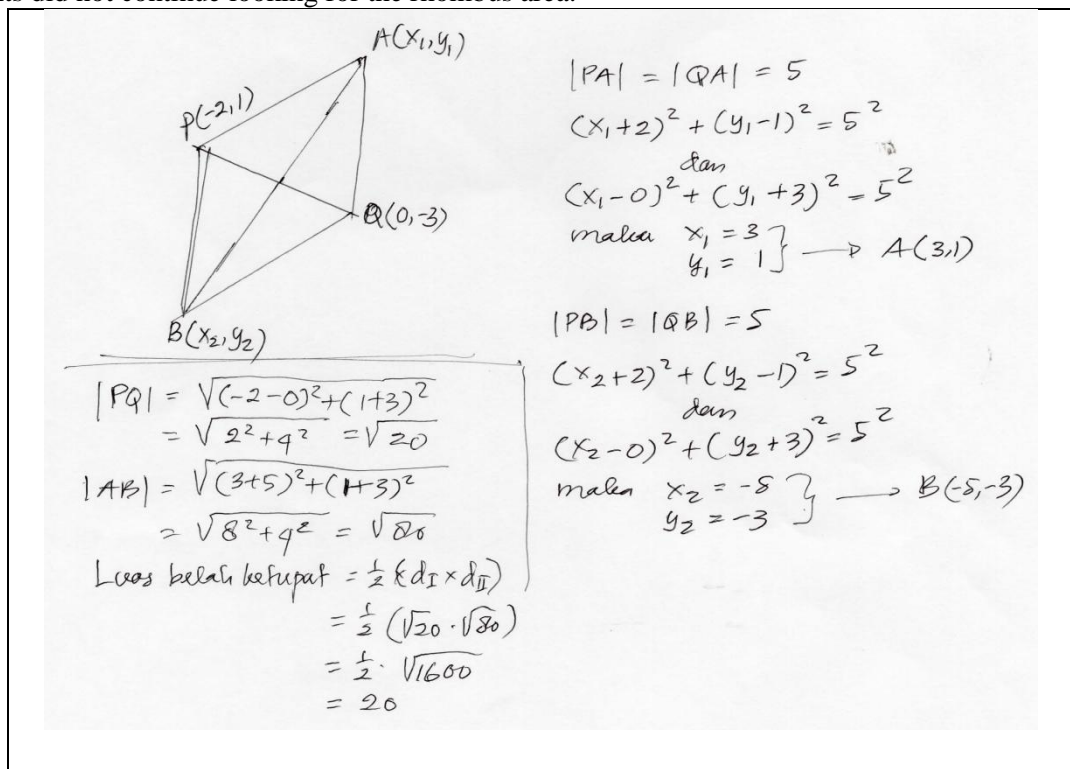


Figure 5. The Resolution Strategy of Visual and Harmonic Students in Solving Rhombus Problem with Paper-pencil

However, students had different strategy in solving the rhombus problem by using the the help of geogebra compared to using paper-pencil. Visual students firstly constructed point P and Q then constructed the PQ segment and looked for the midpoint coordinates of the PQ using the toolbar "midpoint or center" to obtain A (-1,-1). After that they constructed a line g that is perpendicular to the PQ segment through point A using the "perpendicular line" tool. Next they constructed any point B which was located on the line g then build PB and QB segments, they then dragged point B along the g line while looking at the algebra window until PB length equal to 5, which was at point B (3,1) that's the length of the rhombus side (5 units). Next they looked for the reflection of point B by using the "Reflect about line" toolbar to get point B' (-6,-3). Their final step was to search for the rhombus area

by building a rhombus using the "polygon" toolbar through points P, B, Q, and B "so that rhombus area shown in the algebra window (figure 6).

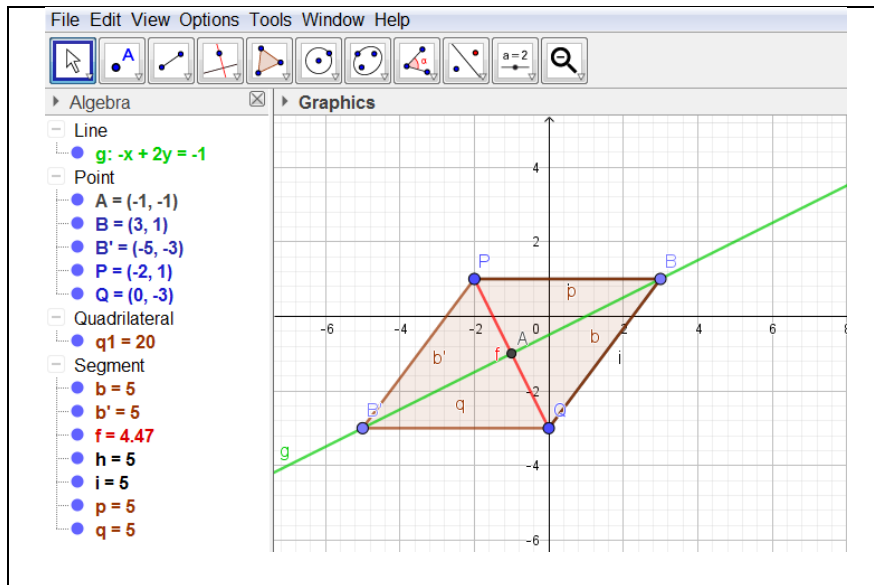


Figure 6. The Resolution Strategy of Visual Student in Solving Rhombus Problem with Geogebra

While harmonic and non-visual students used the following steps: first, they constructed points P and Q, then built a circle with center P and radius 5 and also built a circle with center Q and radius 5. Next they determine the intersection of the two circles which were the remaining vertices of the rhombus, then built the rhombus using the "polygon" toolbar which passes the points P, A, Q, and B so that the rhombus area was found through the algebra window (figure 7).

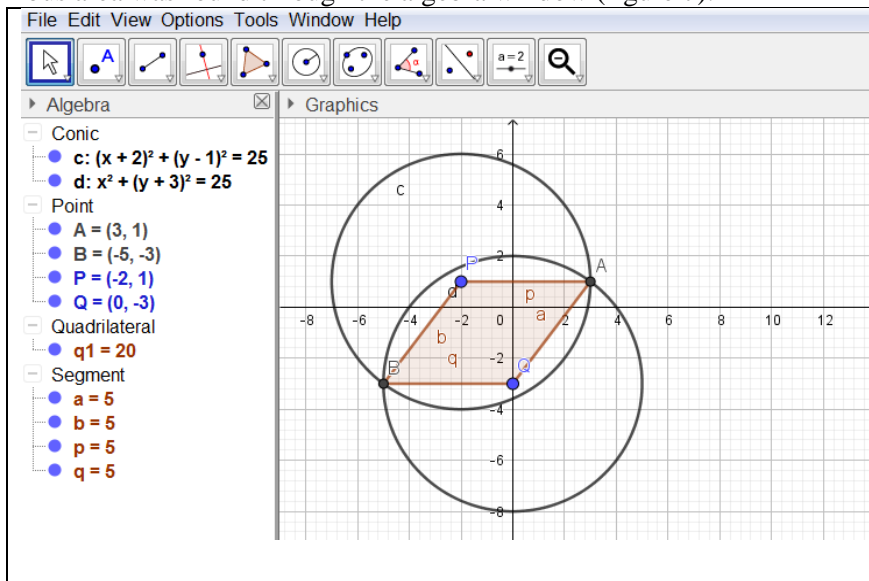


Figure 7. The Resolution Strategy of Harmonic and Non-visual Students in Solving Rhombus Problem with Geogebra

4. Discussion

After looking at all the cases in this study which are visual, harmonic and non-visual students and their performance in finding solution of the problem, researchers found there were three behavioral typologies that emerged from the interpretation of the degree of acquisition of instrumentation and instrumentalization, and connected with the context of this study :

The Instrumentation and instrumentalization degrees of acquisition of the visual students are high. They have well-developed problem solving skills. They are also able to represent geometrical objects and justify their conjectures. Moreover, they don't find difficulty in visualizing, or solving algebraic problems. Geogebra really help and assist them in the process of visualization. However, that does not help them to understand the concepts of the problem. They tend to reason using the picture and they don't have difficulties in using geogebra. The typology of their behavior is *Autonomous* [3, 8].

Meanwhile, harmonic students' instrumentation and instrumentalization acquisition degrees acquisition shows medium to high ability. Moreover, they are good students with few intuitive skills and don't find difficulty when they used geogebra and it helps them to visualize the problem. They are using paper and pencil strategies, which is based on algebraic calculations (analytic strategies), while the use of geogebra fosters more geometrical thinking reasoning on the picture. The typology of their behavior is *Instrumental* [3, 8].

Non-visual students' instrumentation degree is higher than instrumentalization degree. They are analytical students. Although they have some conceptual difficulties such as the idea of distance from point to line, they easily get familiarized with instructions from new concepts. In geogebra's resolution, they base their reasons both on figure and drawing. The typology of their behavior is *Procedural* [3, 8].

Outside the typology of students' mathematical behavior, researchers found other interesting results. For example, researchers have difficulty transferring geogebra's strategy to paper-pencil strategies. Geogebra's dynamic resolution strategy for rhombus problems (Figure 6), based on the diagonal acquisition of the two rhombus and vertices of B and B' remaining, does not have a clear transfer for the resolution of paper and pencil. Students in Figure 6 obtained a rhombus by dragging point B along the diagonal until one of the segments measured in length is 5 units.

Most students use algebra and measuring instruments. The researcher observed, there was a simultaneous use of algebra windows and geometric windows in geogebra resolution. For example, for a circular problem, the harmonic student constructed the third point of circle $P' = S f (P)$ (the center of the circle belongs to the line f , P belongs to the circle so that P' belongs to the circle too). After that, all students used the circle tool through three points to get a circle in the geometric window. Finally, it obtained a radius and center circle coordinates by interpreting the circle equation that appeared in the algebra window: $(x - a)^2 - (y - b)^2 = R^2$. In this case, simultaneous use of algebraic windows and geometric windows, influenced student strategies.

This result supported the finding of Iranzo and Fortuny. He reported that the existence of different acquisition degree of geometrical abilities concerning the students' processes of instrumentation and instrumentalization when they work together in a geogebra and paper-and-pencil media [14]. This suggests that changing the environment may prompt students to seek for additional solutions, which, in turn, results in a deeper understanding of the problem. As such, using both environments simultaneously in solving the same problems appears to bring about important benefits.

5. Conclusion and Suggestion

Based on the result of tests, observation, and interview to the students, the researcher found out that the existence of different acquisition degree of geometrical abilities concerning the students' processes of instrumentation and instrumentalization when they work together in a geogebra and paper-and-

pencil media. Visual students at the autonomous level, harmonic students at the instrumental level, and non-visual students at the procedural level.

The method used to identify and characterize the mathematical behavior of students in this study is based on the interpretation of many data. However, there is data that is still need to be be found out and, on the other hand, the didactical situation is very special, as it has been designed without including time for lecturer instruction. In connection with the introduction of new data, future research must help to form a better understanding of the process using software and to analyze the emergence of joint, connectivity and synergy of computational and paper-pencil techniques to promote argumentative ability in geometry of university level. In connection with the introduction of new didactic situations, future research must include the role of the teacher, together with the interaction of geogebra-students and students-students.

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